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Hadamard speckle contrast reduction

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The conditon for a diffuser to produce the maximum speckle contrast reduction using the minimum number of distinct phase patterns is derived. A binary realization of this optimum diffuser is obtained by mapping the rows or columns of a Hadamard matrix to the phase patterns. The method is experimentally verified in the Grating Light Valve™ laser projection display.

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The use of lasers in a projection display enables the creation of vibrant images with extensive color coverage that is unachievable by conventional sources. One major obstacle to preserving the image quality, which has been well-known since the invention of visible lasers, is a phenomenon called speckle[1]. Speckle arises when coherent light scattered from a rough surface, such as a screen, is detected by a square-law detector with a finite aperture, such as an observer's eye. The image on the screen appears to be quantized into areas with sizes equal to the detector resolution spot. The detected spot intensity varies randomly from darkest, if contributions of the scattering points inside the spot interfere destructively, to brightest if they interfere constructively. This spot-to-spot intensity fluctuation is referred to as speckle.

In a laser projection display, and in coherent imaging systems in general, the presence of speckle tends to mask the image information; therefore the reduction of speckle is highly desirable. Following Goodman[2,3], speckle contrast will be used as a measure of speckle. It is defined as the ratio of the standard deviation to the mean of the speckle intensity, and its value is 0–1. Speckle reduction is based on averaging M independent (i.e. uncorrelated and noninterfering) speckle configurations within the spatial and temporal resolutions of the detector. Goodman[2,3] has proven that, under the most favorable condition in which all the M independent speckle configurations have equal mean intensities, the configurations add on an intensity basis and the speckle contrast is reduced from 1 to $M^{-1/2}$. Fully coherent configurations, on the other hand, add on an amplitude basis and the speckle contrast is unreduced.

One common approach[4,5] to generating multiple speckle configurations is to superimpose upon the amplitude image a time-varying diffuser, usually placed at an intermediate image plane. The role of the diffuser is to partition each detector-resolution spot into M cells and to assign each cell a phase. M is usually taken to be as large as permitted by the imaging optics' numerical aperture (NA) because the minimum cell size ~ λ/NA. The set of the M phase cells that covers one resolution spot constitutes a phase pattern. Temporally varying the phase pattern faster than the detector's temporal resolution will effectively destroy the spatial coherence of the light coming from the phase-cells, thereby reducing the speckle contrast. If the maximum reduction, $M^{1/2}$, is achieved with the minimum number of distinct phase patterns, M, the speckle reduction is referred to as optimum. The purpose of this Letter is to derive the condition for the optimum diffuser and to present a binary realization.

Let a square (so called for mathematical simplicity) detector-resolution spot be divided into $M = N_1N_2$ equal cells arranged in N_1 rows and N_2 columns, as shown in Fig. 1(a). If the detected optical field from the ij^{th} cell on the screen is E_{ij}, where $i = 1, 2, \ldots, N_1$ and $j = 1, 2, \ldots, N_2$, the speckle intensity of the resolution spot is

$$I_0 = \left| \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} E_{ij} \right|^2 . \tag{1}$$

The fields add together on an amplitude basis, and the speckle contrast remains unreduced. Here the speckle contrast is evaluated over an ensemble of resolution spots. Suppose a diffuser that imprints M phase cells with phase $\phi^{a}{}_{ij}$ is superimposed upon the original resolution spot, as shown in Fig. 1(b). Suppose further that A different phase patterns are sequentially presented with equal duration during the detector's integration time; then the speckle intensity becomes

$$I = \frac{1}{A} \sum_{a=1}^{A} \left| \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} h_{ij}^{a} E_{ij} \right|^2 , \tag{2}$$

where $h^{a}{}_{ij} = 1 \cdot \exp(i\phi^{a}{}_{ij})$. If the summation of $h^{a}{}_{ij}$ over all the A phase patterns satisfies

$$\sum_{a=1}^{A} h_{ij}^{a^*} h_{kl}^{a} = A\delta_{ik}\delta_{jl} \quad , \tag{3}$$

then

$$I = \frac{1}{A}\sum_{a=1}^{A}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_1}\sum_{l=1}^{N_2} h_{ij}^{a^*} E_{ij}^{*} h_{kl}^{a} E_{kl} = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\left|E_{ij}\right|^2 \quad . \tag{4}$$

The averaging forces the cross-terms to vanish. The M cells decorrelate from each other, and their contributions become independent. Unlike in Eq. (1), the fields now add together on an intensity basis, and therefore the speckle contrast is reduced by a factor of $M^{1/2}$. This reduction is maximum because the upper limit of independent configurations that can be generated is M. It is also clear that the number of phase patterns to produce M independent speckles cannot be fewer than $A_{min} = M$. In fact, the traditional random diffuser needs a large number (theoretically infinite) of phase patterns to reach the $M^{1/2}$ reduction[4]. Therefore the set of phase patterns that produces the optimum speckle reduction must satisfy

$$\sum_{a=1}^{M} h_{ij}^{a^*} h_{kl}^{a} = M\delta_{ik}\delta_{jl} \quad . \tag{5}$$

Next it will be shown that a family of binary phase patterns, derived from the rows or columns of a Hadamard matrix[6], can be used to achieve Eq. (5). Binary phase patterns have the advantage of simple hardware implemetation. A Hadamard matrix of order M, denoted by $H(M)$, is an $M \times M$ matrix with ±1 entries that satisfies

$$H^T(M)\,H(M) = M\,I(M) \quad , \tag{6}$$

where $I(M)$ is the $M \times M$ identity matrix. The Hadamard matrix exists for $M = 2^{\text{integer}}$, and likely also for $4 \times$ integer. The Hadamard matrix has the following properties: (a) any two rows or two columns are orthogonal, (b) permutations of rows or columns preserve Eq. (6), and (c) reversing the sign of a row or a column preserves Eq. (6). For $M = 2^{\text{integer}}$, the Hadamard matrices can be generated recursively by Sylvester construction:

$$H(1) = 1 \qquad\qquad H(2M) = \begin{pmatrix} H(M) & H(M) \\ H(M) & -H(M) \end{pmatrix} \quad . \tag{7}$$

Let h^a be an $N_1 \times N_2$ phase pattern and H a Hadamard matrix of order $M = N_1N_2$. Consider the 1-1 mapping that takes the ab^{th} element of H, where $b \equiv (i\text{-}1)N_2 + j$, to the ij^{th} element of h^a according to:

$$h^a_{ij} = H_{a,(i-1)N_2+j} \quad , \tag{8}$$

as illustrated in Fig. 2 (the column mapping proceeds in an analogous manner). The +1 and –1 entries of the Hadamard matrix correspond to $\exp(i0) = +1$ and $\exp(i\pi) = -1$ cells, respectively, of the phase pattern. By use of column orthogonality property (a), $\Sigma_a H_{ab}H_{ac} = M\delta_{bc}$, and by the fact that $(k\text{-}1)N_2 + l \equiv c = b$ if and only if $k = i$ and $l = j$, it follows that:

$$\sum_{a=1}^{M} h^a_{ij} h^a_{kl} = \sum_{a=1}^{M} H_{a,(i-1)N_2+j} H_{a,(k-1)N_2+l} = M\delta_{ik}\delta_{jl} \quad . \tag{9}$$

This proves that the set of real phase patterns $\{h^a, a = 1, 2, \ldots, M\}$ satisfies the optimum speckle reduction condition in Eq. (5). The result is obviously not unique, as properties (b) and (c) can be used to generate different Hadamard matrix representations. This freedom can be exploited in practice to create other sets of phase patterns that meet additional criteria, such as patterns that possess a certain symmetry or patterns that produce specific light distribution at the pupil plane. Also, the result applies for other phase pattern geometries (e.g., a hexagon with triangular phase cells).

To illustrate the above results we consider the situation when $N_1 = N_2 = 4$ case. The Sylvester-type $H(16)$ is economically displayed in Fig. 3 as a so-called batik pattern, in which a white cell stands for +1 and a black cell for −1. Applying Eq. (8) yields the sixteen 4×4 phase patterns given in Fig. 4(a). Figure 4(b) shows another valid set of phase patterns, obtained by reversal of the sign of the sixth column of $H(16)$.

The Hadamard speckle contrast reduction was tested in the Grating Light Valve™ (GLV) laser projection display[7,8]. The GLV is a one-dimensional spatial light modulator with 1080 pixels. One forms a two-dimensional image is formed by scanning the one-dimensional image across the screen while modulating the GLV with successive column information. The speckle contrast measurement was standardized, so the detector-resolution spot was about one GLV pixel on the screen. The maximum reduction permitted by the imaging optics' NA and the GLV pixel size was $M^{1/2} \approx 8$, which prompted the use of $H(64)$ in a representation with sixty-four modulo 8 cyclic phase patterns. The diffuser was made by etching of the phase patterns in a fused-silica wafer by use of a single mask in standard lithography fabrication. For a π phase-shift the etch depth was $\lambda/2(n\!-\!1) = 577$ nm ($\lambda = 532$ nm, $n = 1.46$). The diffuser was placed at a plane conjugate to the GLV array and was set in transverse oscillatory motion by a voice coil. All the phase patterns must be presented within the detector integration time; this was accomplished by combining the scanning action and the diffuser transverse motion across the eight noncyclic phase patterns.

A CCD camera that operates in the linear regime was used to capture the speckle images. Each speckle image was normalized to eliminate any background contribution. The original speckle contrast, shown in Fig. 5(a), was measured to be 0.70, close to the expected value of $1/\sqrt{2}$ from a single-spatial-mode narrow-band laser scattered off a depolarizing screen. The reduced speckle contrast in Fig. 5(b) was 0.09, which is in good agreement with the calculated result of 0.70/8. The original image quality was largely preserved by the diffuser. The measured optical efficiency, taken as the ratio of the light power transmitted to the screen with and without the diffuser, was 85%. The combination of Hadamard and other speckle contrast reduction methods is described in Ref. 8.

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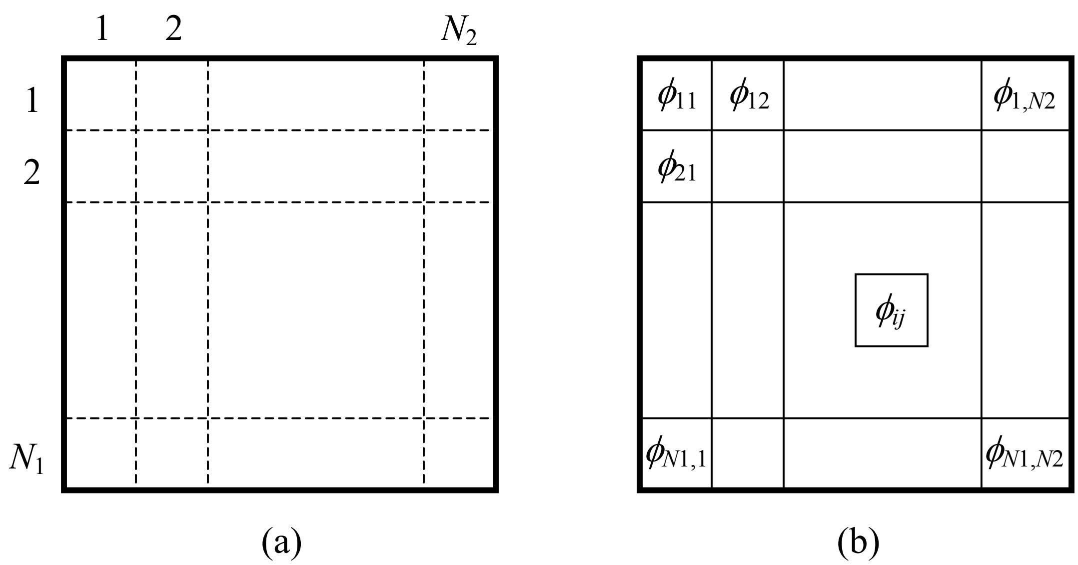


Fig.1. (a) Detector resolution spot partitioned into $M = N_1N_2$ cells, (b) phase pattern with M phase cells superimposed upon the resolution spot.

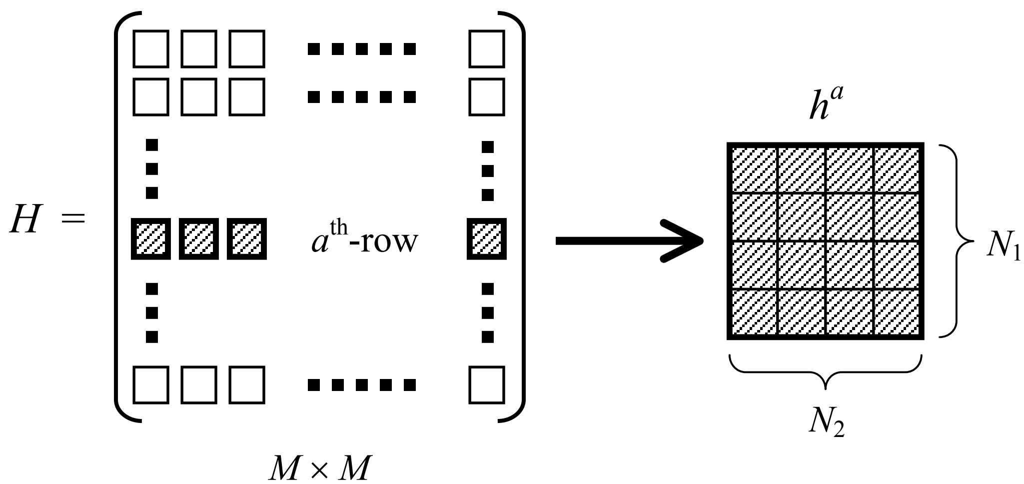


Fig. 2. Mapping from the a^{th} row of H to the a^{th} phase pattern.

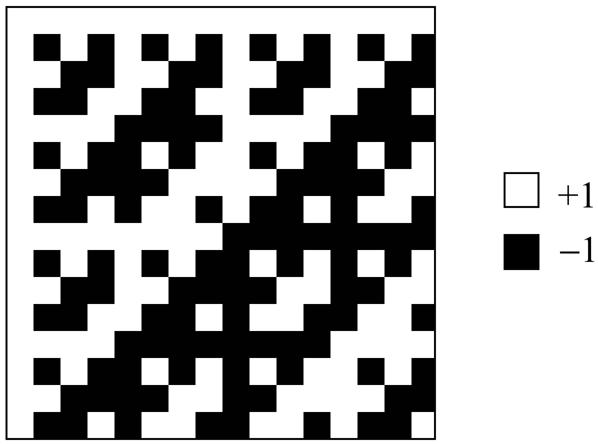


Fig. 3. Batik pattern of Sylvester-type $H(16)$.

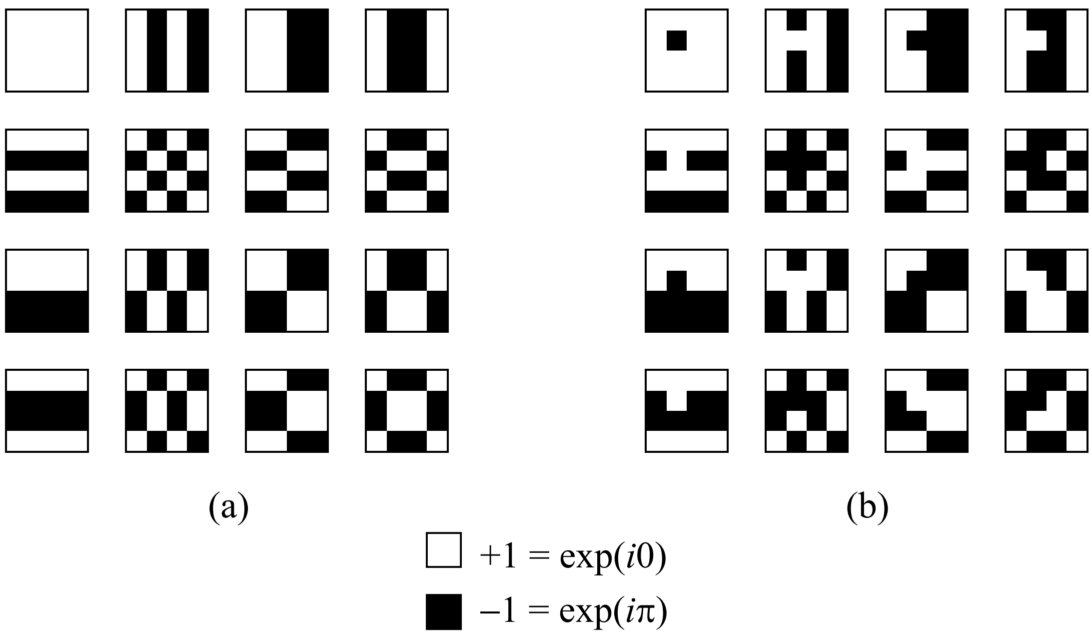
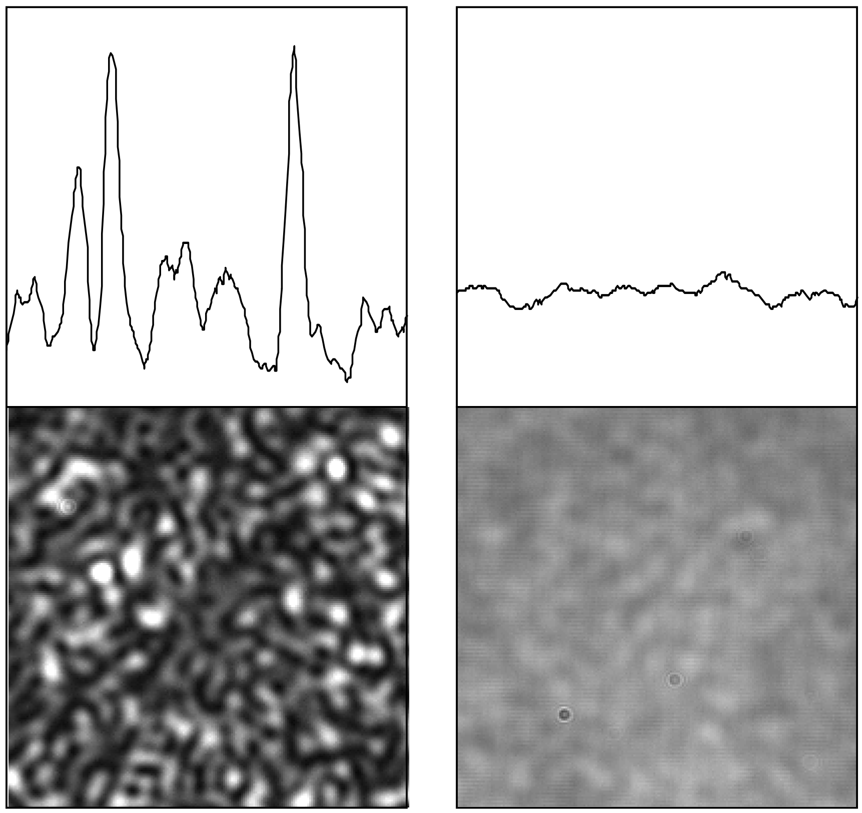


Fig. 4. (a) Sixteen batik phase patterns from the Sylvester-type $H(16)$, (b) the effect of negating the sixth column of $H(16)$.



(a) (b)

Fig. 5. Images and samples of intensity fluctuation of (a) the original speckle and (b) the reduced speckle.